Regional Attenuation Models in Central and Eastern North America Using the NGA-East Database

Abstract:
The anelastic attenuation term found in Ground-Motion Prediction Equations (GMPEs) represents the distance dependence of the effect of intrinsic attenuation upon the wavefield as it propagates through the crust, and contains the frequency-dependent quality factor, Q(f), which is an inverse measure of the effective anelastic attenuation. In this work, regional estimates of Q(f) in Central and Eastern North America (CENA) are developed, building off the recent work by others including NGA-East. The technique employed uses smoothed Fourier amplitude spectrum (FAS) data from well-recorded events in CENA as collected and processed by NGA-East. Regional Q(f) is estimated using an assumption of average geometrical spreading applicable to the distance ranges considered, a correction for the radiation pattern effect, and a correction for site response based on Vs30. Apparent Q(f) estimates from multiple events are combined within each region to develop the regional models. Models are provided for three NGA-East regions: The Gulf Coast, Central North America, and the Appalachian Province. There was not sufficient data to adequately constrain the model in the Atlantic Coastal Plain region. There is general agreement that tectonically stable regions are usually described by higher Q(f) and weaker frequency dependence (η), while active regions are typically characterized by lower Q(f) and stronger frequency dependence, and the presented models are consistent with these expectations. Overall, the results support the NGA-East regionalization, as significantly different regional Q(f) values for particular events with data recorded in multiple regions are identified. The models for all three regions are valid for use in CENA based on a literature review and a comparison with previously published models.

Note: The following files were submitted by the author for peer review, but cannot be converted to PDF. You must view these files (e.g. movies) online.

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Regional Attenuation Models in Central and Eastern North America Using the NGA-East Database

Jeff Bayless

The anelastic attenuation term found in Ground-Motion Prediction Equations (GMPEs) represents the distance dependence of the effect of intrinsic attenuation upon the wavefield as it propagates through the crust, and contains the frequency-dependent quality factor, $Q(f)$, which is an inverse measure of the effective anelastic attenuation. In this work, regional estimates of $Q(f)$ in Central and Eastern North America (CENA) are developed, building off the recent work by others including NGA-East. The technique employed uses smoothed Fourier amplitude spectrum (FAS) data from well-recorded events in CENA as collected and processed by NGA-East. Regional $Q(f)$ is estimated using an assumption of average geometrical spreading applicable to the distance ranges considered, a correction for the radiation pattern effect, and a correction for site response based on $V_{s30}$. Apparent $Q(f)$ estimates from multiple events are combined within each region to develop the regional models. Models are provided for three NGA-East regions: The Gulf Coast, Central North America, and the Appalachian Province. There was not sufficient data to adequately constrain the model in the Atlantic Coastal Plain region. There is general agreement that tectonically stable regions are usually described by higher $Q(f)$ and weaker frequency dependence ($\eta$), while active regions are typically characterized by lower $Q(f)$ and stronger frequency dependence, and the presented models are consistent with these expectations. Overall, the results support the NGA-East regionalization, as significantly different regional $Q(f)$ values for particular events with data recorded in multiple regions are identified. The models for all three regions are valid for use in CENA based on a literature review and a comparison with previously published models.

Keywords
ground motion attenuation, quality factor, CENA, stochastic method, seismic hazard
INTRODUCTION

In tectonically active regions of the United States, such as California, the seismicity rates are sufficient such that design ground motions can be estimated using empirical ground motion prediction equations (GMPEs, also called ground motion models, GMMs). However, for areas with low rates of seismicity, such as Central and Eastern North America (CENA) it is challenging to develop empirical GMPEs because very few data exist, and most are for small magnitude earthquakes. Although they are infrequent, the potential for large earthquakes exists in CENA, therefore, developing GMPEs for this region requires alternative methods beyond empirical modeling. Substantial effort has been made on this topic, including Boore and Atkinson (1987), EPRI (1993), Toro et al. (1997), Silva et al. (2002), Abrahamson and Silva (2001), Atkinson (2012), and most recently, the collaborative effort of the Pacific Earthquake Engineering Research Center’s NGA-East (PEER, 2015; NGA-East hereafter).

When deriving GMPEs in data-poor regions, several alternatives exist, but earthquake simulations are widely used for supplementing the recorded data. Over the last several decades, the stochastic point-source method has been the commonly used simulation method for this purpose. The stochastic method is based on the pioneering work of Brune (1970), Hanks and McGuire (1981) and Boore (1983), among others. David Boore extended it to the simulation of acceleration time series in Boore (1983) and Boore (2003). Following Boore (2003), a simulated time series is produced using a seismological model of the Fourier amplitude spectrum, and assuming the spectrum is distributed with random phase angles over a time duration related to the earthquake magnitude and the distance between the source and site. For more details on the stochastic method, the reader is referred to Appendix 3A of the NGA-East Report (PEER, 2015), or to Boore (2003), both of which give excellent descriptions.

Details in the application of this method vary, but the conventional stochastic method uses an omega-square source model (Brune 1970) with a single-corner frequency and a constant stress drop (Boore 1983; Atkinson 1984), in which the shape of the acceleration FAS spectral density $Y$ at frequency $f$ is given by Equation 1,

$$Y(f) = \hat{\gamma} \cdot \frac{f^2}{1 + \left( \frac{f}{f_0} \right)^2} M_0 A(f) D(f) G(R) \exp \left( -\frac{\pi f R}{Q(f) \beta_0} \right) \#(1)$$
in which $f$ is the frequency, $M_0$ is seismic moment, $R$ is the effective source-site distance, $f_0$ is the source corner frequency (related to the seismic moment and stress drop, $\Delta \sigma$), $\beta_0$ is the shear-wave velocity near the source, $A(f)$ is the crustal amplification, $D(f)$ is the high-frequency diminution term (kappa term), $G(R)$ is the geometric spreading function ($1/R$ for a uniform whole space), and $\hat{C}$ is a constant that accounts for the source region material density, the effect of the free surface, averaged source radiation pattern, and the partition of energy into two horizontal components. The final exponential term represents the distance dependence of the effect of intrinsic attenuation upon the wavefield as it propagates through the crust. The quality factor, $Q(f)$ is an inverse measure of effective anelastic attenuation, which introduces a decay in spectral amplitudes with distance; this attenuation is frequency dependent, and thus alters spectral shape (Atkinson and Boore, 2014). The purpose of this study is to develop improved regional estimates of $Q(f)$ in CENA.

Taking the natural logarithm ($ln$) of both sides of Equation 1 and using the product rule of logarithms yields Equation 2. The form of this equation resembles the basic form of many GMPEs for median response spectra, $PSA_{med}(f)$, e.g. Equation 3, where the related quantities in Equations 2 and 3 are aligned on the page. Source is a collection of earthquake source-related terms generally described by moment magnitude ($M$) and style of faulting, Site is a collection of site amplification terms (often parameterized by $V_{s30}$ and basin depths), $b$ is the frequency-independent geometric spreading coefficient, and $c(f)$ is the coefficient of anelastic attenuation. From Equations 2 and 3, the relationship between $c(f)$ and $Q(f)$ is given by Equation 4.

$$
\ln[Y(f)] = \ln[\hat{C} \frac{f^2}{1 + \left(\frac{f}{f_0}\right)^2}M_0] + \ln[A(f)D(f)] + \ln[G(R)] + \left(\frac{-\pi f R}{Q(f)\beta_0}\right) \#(2) 
$$

$$
\ln[PSA_{med}(f)] = Source(f) + Site(f) + b \ln[G(R)] + c(f) R \#(3) 
$$

$$
Q(f) = \frac{-\pi f}{c(f)\beta_0} \#(4) 
$$

$Q(f)$ is believed to be attributable to intrinsic absorption, plus the frequency-dependent effects of scattering (Dainty 1981; Atkinson, 2012) and is usually modeled with the form $Q(f) = Q_0 f^\eta$.
where $Q_0$ is the $Q$ value at 1 Hz, and $\eta$ is the slope parameter. The geometric attenuation ($b$ term) models the amplitude decay due to the expanding surface area of the wave front as it propagates away from the source, and generally controls the attenuation of ground motions at near source distances. At distances greater than about 100 km, the anelastic attenuation effects become dominant (Atkinson, 2012). This is evident in Equation 3, for which the geometric spreading attenuation scales with $\ln(R)$, and the anelastic attenuation scales with $R$. However, the geometric spreading and anelastic attenuation are coupled, and empirical studies have shown that the same data can be fit (for particular $M$ and distance ranges) with different trade-offs between parameters $b$ and $c$. Therefore, suites of parameters developed empirically are relative to each other, and care must be taken when separately evaluating one term from one study, with another from another study. For this reason, $b$ is fixed at -0.5, which corresponds to the theoretical value for surface waves in a half-space and is a generally agreed upon value in eastern North America at regional distances (Atkinson and Boore, 2014). This selection is supported further in the Approach section of this paper.

**Previous Work**

Previous studies of regional CENA attenuation models are numerous; recent works include Cramer et al. (2012), Cramer and Al-Noman (2014), Deshon and Bisrat (2010), Sandoval (2014), and NGA-East (PEER, 2015). The NGA-East project was a multi-disciplinary research project managed by Christine Goulet with the objective to develop a new ground motion characterization (GMC) model for CENA. Part of this project was to develop median GMPEs for the region, and this task included eight categories of approaches, split into ten chapters, with a different GMPE (and authors) for each chapter. Six of these chapters utilize some variation of stochastic method modeling, and therefore have either adopted or inverted models for $Q(f)$. As a starting point for NGA-East, Boore (2015) compiled attenuation (geometric spreading and anelastic attenuation) models from the literature; over 40 were identified and after review, six high quality models were selected to span the range of available models while maintaining a manageable number of models. These are summarized in Table 2.1 of Boore (2015). The attenuation models (geometrical spreading and anelastic attenuation) used with stochastic method simulations as part of the NGA-East project are used as the basis for comparisons in the section titled Model Comparison.
Additionally, Cramer (2012) studied boundaries between major $Q$ regions in the continental US using the Earthscope USArray data. This was accomplished using transects of observations across the transitions to look for major changes in $Q$. In this process, Cramer (2012) determined regional estimates of apparent $Q$; these are also used as the basis for comparisons in this paper. The estimates of $\eta$ from Cramer (2012) range from 0.5 to 0.8. These values are generally larger than those used in the NGA-East project, which range from 0.3 to 0.64, with the exception of Silva et al. (2001) which uses $\eta = 0.84$.

**APPROACH**

Estimating $Q(f)$ requires the knowledge of a large number of parameters including source terms, geometrical spreading, and receiver terms. A more reliable $Q(f)$ model is obtained when the size of the problem is minimized by imposing constraints on some of these parameters. The technique adopted includes collecting data from well-recorded events in CENA and estimating regional $Q(f)$ using (1) an assumption of average geometrical spreading, (2) a correction for the radiation pattern effect, and (3) a correction for site response based on $V_{s30}$, the time-averaged shear wave velocity in the upper 30 meters of the soil column at the site. This approach is described under the following sub-headings: Ground-Motion Data, Data Selection, Inversion for $Q$. 

*Ground Motion Data*

The database utilized is a subset of the PEER NGA-East database compiled and processed by Goulet et al. (2014). As described in Goulet et al. (2014), this database includes events with $M > 2.5$, at distances up to 1500 km, recorded in CENA since 1988. The final NGA-East database contains over 29,000 records from 81 earthquake events and 1379 recording stations. As is standard with all PEER NGA projects, the time series and metadata went through numerous rounds of quality assurance and review.

The ground-motion parameter used in the analysis is the smoothed $EAS$, as defined by and used in the PEER NGA-East GMPE (Hollenback et al., 2015). The $EAS$ is the orientation-independent horizontal component $FAS$ of ground acceleration. The $EAS$ is calculated for an orthogonal pair of $FAS$ using Equation 5,
\[
EAS(f) = \frac{1}{\sqrt{2}} [FAS_{HC1}(f)^2 + FAS_{HC2}(f)^2]^{\#(5)}
\]

where \( FAS_{HC1} \) and \( FAS_{HC1} \) are the FAS of the two as-recorded orthogonal horizontal components of the ground motion and \( f \) is the frequency in Hz. The FAS are processed by PEER following the procedure given by Kishida et al. (2016). The \( EAS \) is independent of the orientation of the instrument. Using the average power of the two horizontal components leads to an amplitude spectrum that is compatible with the use of RVT to convert Fourier spectra to response spectra. To maintain consistency with other PEER ground-motion studies, the \( EAS \) is smoothed using the log_{10}-scale Konno and Ohmachi (1998) smoothing window, with smoothing parameters described by Kottke et al. (2018).

Several previous studies on \( Q(f) \) have used the vertical component of FAS; this is often because the vertical component data are most plentiful, but this requires using H/V ratios to estimate horizontal ground motions attenuation. Using vertical records is considered acceptable because other studies that have shown that there are no apparent differences between horizontal and vertical component attenuation over the distance range 100-800 km (Atkinson 2012). However, by using horizontal component ground motions directly this additional step is avoided.

The NGA-East project also included a working group focused on regionalization (Dreiling et al. 2014). This effort divided CENA into four regions based on the geologic and tectonic setting. These regions are shown below in Figure 1 (reproduced from Goulet et al., 2015; Figure 1.2). The regions are numbered as: (1) the Gulf Coast, (2) Central North America (CNA), (3) the Appalachian Province, and (4) the Atlantic Coastal Plain.
Figure 1. Reproduction of Goulet et al. (2015) Figure 1.2, showing the four CENA regions: (1) the Gulf Coast, (2) Central North America (CNA), (3) the Appalachian Province, and (4) the Atlantic Coastal Plain.

Data Selection

The NGA-East database (Goulet et al. 2014) products include a “flatfile” with recording metadata and response spectra, time series files, and FAS files. Christine Goulet provided us with a flatfile including the EAS (personal communication, 2019). In this file, the EAS has been calculated for each record in the database up to the Nyquist frequency by PEER following the Kishida et al., (2016) processing method. The lowest and highest usable frequencies of each record are determined following Abrahamson and Silva (1997). By limiting the usable period range, the frequency interval of the impulse response of a 5% damped oscillator will not exceed the filter values. Further, retaining the Abrahamson and Silva (1997) usable frequency range maintains consistency with response spectrum models.

For each event, the subset of data with rupture distances between 150 and 500 km is selected. The data at distances smaller than 150 km, for which the onset of critical reflections from the lower crust may be important (Burger et al., 1987; Somerville et al., 1990), are excluded so that the geometric spreading assumption \((b = -0.5)\) is appropriate; this is also consistent with the models given in Boore (2015). The upper limit of 500 km was selected so that the regional effects of the apparent anelastic attenuation can be observed, and also to reduce the amount of noise in the data. Atkinson and Boore (2014) used the same distance range for studying attenuation in eastern North America and found that similar results were found for the 200 to
600 km distance range, but with a tendency for slightly more gradual attenuation rates as the
distance range is moved towards larger distances.

In addition to the quality assurance and review performed by PEER (2015), each EAS are
visually checked for outliers, poor quality data, or errors with units. After screening for data
quality, recording distance, recording coverage, and frequency limitations, 53 earthquakes are
identified as candidates for the analysis, each with at least 5 ground motion recordings. These
earthquakes and their attributes are listed in Table A-1 of Appendix A. Figure 2 shows a
magnitude versus rupture distance scatterplot of this database at \( f = 1 \) Hz, and a map of these
events along with the recording stations used in the analyses. This database encompasses over
2,000 EAS records from the 53 earthquakes. The regional \( Q(f) \) estimates are derived from
subsets of this database.

Inversion for \( Q \)

Several studies (e.g Chapman and Godbee, 2012; Frankel, 2015; Graves, 2013) have shown
that radiation pattern and rupture directivity are important factors in determining the
attenuation of ground motions (rate of decrease of ground motion amplitudes with distance),
and that low-frequency amplitudes (in some cases up to 5 Hz) can be contaminated by radiation
pattern and directivity effects. Consequently, it is preferable to take these factors into account

Figure 2. Left: Magnitude versus distance coverage of the data used in the \( Q(f) \) analyses, at \( f = 1 \) Hz.
Right: Map of epicenters for the events used (red stars), along with all recording stations (green triangles)
with data available at \( f = 1 \) Hz. The [2, 4] region boundaries are given by the black and magenta lines,
respectively.
when constructing GMPEs and $Q(f)$ models. The procedure taken to estimate the apparent $Q(f)$ for a given earthquake is as follows:

1. Gather the $EAS$ data and metadata. Filter by region as needed. The unmodified data is denoted $EAS_{raw}$.

2. Calculate the site response adjustment for each record, $F_{Site}$, as described below.

3. Calculate the radiation pattern effect adjustment for each record, $F_{Rad}$, as described below.

4. Adjust the $EAS_{raw}$ for site effects (to obtain $EAS_{Site} = EAS_{raw}/F_{Site}$), for radiation pattern effects (to obtain $EAS_{Rad} = EAS_{raw}/F_{Rad}$), and for both effects to obtain $EAS_{RadSite} = EAS_{raw}/(F_{Site}F_{Rad})$.

5. Follow the Cramer (2012) procedure for estimating apparent $Q(f)$. Assuming $1/\sqrt{R}$ geometrical spreading, fit the attenuation of the $EAS$ at frequency $f$ to Equation 6,

$$\ln[EAS(f)] = A(f) + b \ln[R_{rup}] + c(f)R_{rup} \quad (6)$$

where $A(f)$ is a regression constant, $b = -0.5$, $R_{rup}$ is the closest distance to the rupture, and $c(f)$ is the apparent anelastic attenuation coefficient.

6. Estimate the apparent $Q(f)$ from $c(f)$ by the relationship given in Equation 4. $\beta_0$ is estimated for each event by interpolating the CENA 1D crustal model from Darragh et al. (2015) Table 3.2 for the shear wave velocity at the hypocentral depth. As in Atkinson and Boore (2014), the constraint that $c(f)$ must be negative is imposed; this corresponds to downward curvature of the attenuation of ground motions with distance. In cases where the range of mean $c(f)$ plus and minus one standard error contained positive estimates (corresponding to flat, or upward curvature of attenuation) this frequency was excluded from subsequent analyses.

This process is repeated for each earthquake in the dataset for 10 log-spaced frequencies ranging from to 1 to 20 Hz, and for each of $EAS_{raw}$, $EAS_{Site}$, $EAS_{Rad}$, and $EAS_{RadSite}$. These four variations of the ground motions are analyzed to assess the effectiveness of the site and radiation pattern corrections on apparent $Q(f)$ estimates. This effectiveness is quantified.
through analysis of the residual standard deviations ($\sigma_{Resid}$) and the standard error of the $c$
coefficient estimates ($se_c$) in the Results section.

The frequency dependence of $Q(f)$ is then fit to the form $Q(f) = Q_0 f^n$. Both this fit and the
fit in Equation 6 are performed using an iteratively re-weighted least-squares regression with
Huber weighting and outlier detection (Holland and Welsch, 1977). In Huber weighting, observations with small residuals get a weight of one and observations with larger residual are assigned reduced weights, and the estimating equation is solved iteratively for the coefficients until convergence. The apparent $Q(f)$ estimates using this procedure are whole record
estimates, which at regional distances from shallow events are dominated by the $L_g$ phase
(mixed with other phases); primarily composed of $S$ waves trapped within the lower seismic
velocities in the crust (Kennett, 1986). Therefore, the results presented here are compatible
with other studies to determine frequency-dependent $L_g$ attenuation in CENA.

*Site Response*

For the site response adjustment (Inversion Step 2) three existing linear models are considered.
The first is the Harmon et al. (2019) linear model, which is developed specifically for smoothed
$FAS$ in CENA. This model was developed from a parametric study of 1D ground response
analyses of input rock motions propagated through soil columns representative of CENA site
conditions using the software DEEPSOIL V6.1 (Hashash et al. 2016). The Harmon et al. (2019)
linear $FAS$ model is in the form of tabulated ln(amplification) for a set of $V_{s30}$, ranging from
90 to 3000 m/s, and $f$, ranging from 1 to 100 Hz. Interpolation is performed of the
ln(amplification) for the $V_{s30}$ of the site and for the frequency under consideration.

The second model considered is from Stewart et al. (2017), which as part of the NGA-East
project, synthesized relevant research results to provide recommendations to the USGS for the
modeling of ergodic site amplification in CENA for application in the next version of USGS
maps. This panel recommended a model composed of three terms; the component used here is
the linear site amplification term which describes $V_{s30}$ scaling relative to a 760 m/s reference
condition. This model is largely empirical, although it is designed for use with 5% damped
pseudo-spectral acceleration instead of $FAS$. Because this model is based on data in CENA it
is retained as one of the options. This model is applicable for $V_{s30}$ from 200 to 2000 m/s and $f = 0.2$ to 12.5 Hz.

The third model considered is from Bayless and Abrahamson (2019), which is an empirical EAS ground motion model developed for California. One component of this ground motion model is an empirical, $V_{s30}$ and frequency based linear site amplification term. This model is applicable for $V_{s30}$ from 180 to 1500 m/s and $f = 0.1$ to 24 Hz. The drawback of using this model is that it is derived from data recorded in California and Nevada, which is well-known to have different geologic conditions than CENA. However, this model is also tested because it is appropriate for correcting the smoothed EAS used in this study.

The effectiveness of these models in estimating apparent $Q(f)$ is quantified through reductions in $\sigma_{\text{Resid}}$ and $se_c$ relative to the uncorrected data, which imply that the attenuation of the data are fit better after applying the site correction. Ultimately, the best performing site response model varies between regions. In the Gulf Coast and Appalachian Province (Regions 1 and 3), the Stewart et al. (2017) site amplification model performs best, and in CNA (Region 2) the Harmon et al. (2019) model performs best. Therefore, these two site response models are adopted for these corresponding regions for the remainder of the study. The comparison of $\sigma_{\text{Resid}}$ and $se_c$ reductions for the three site response models are given in Appendix C.

**Radiation Pattern**

The radiation pattern adjustment (Inversion Step 3) is based on 2-dimensional estimations obtained by averaging the 3-dimensional radiation amplitude pattern focal sphere for S waves (Equations 4.84 and 4.85 from Aki and Richards, 1980) over a narrow range of azimuths and take-off angles for a specific focal mechanism and source-receiver azimuth. The take-off angle is randomized around 30 degrees (measured from horizontal) for high frequencies, where the randomization becomes narrower as the frequencies approach 1 Hz. Following Boore and Boatwright (1984), this takeoff angle falls within the recommended range for regional source to site distances. Here S represents the total S motion ($= \sqrt{SH^2 + SV^2}$). For a given azimuth, the radiation coefficient is normalized by the average over the whole focal sphere. Using this procedure results in a dimensionless radiation amplitude pattern parameter that varies with
azimuth, given the earthquake strike, rake, dip. In most cases, the radiation pattern adjustment falls within a factor of 2.

The four-lobed apparent radiation pattern is expected to be gradually distorted with increasing frequency (Takemura et al. 2016). To model the saturation of radiation pattern with increasing frequency, the procedure of Pitarka et al. (2001) is followed in which, at higher frequencies, the 2D radiation pattern is washed out and becomes a circle (independent of azimuth) at 3 Hz. The $\sigma_{\text{Resid}}$ and $se_c$ reductions after accounting for site and radiation pattern effects are given in Appendix C.

Figure 3. Left: A map showing the M4.7 Sparks earthquake epicenter (red star) and recording stations in Region 1 (green triangles) used in the inversion. The 2-dimensional S radiation pattern at $f = 1.5$ Hz is shown by the dashed line, for earthquake strike, rake, and dip of 300, 80, and -10 deg, respectively. Top Right: Azimuthal variation of the radiation pattern adjustment, for the M4.7 Sparks earthquake. The small black symbols are the 2-dimensional estimations of the total S motion radiation amplitude pattern normalized by the averaged over the whole focal sphere. Red circles are the recording stations. Bottom Right: Attenuation with distance at $f = 1.5$ Hz of the data ($EAS_{\text{RadSite}}$), along with the mean fit of Eq. 10 (red), plus and minus one standard deviation. The black curve is the geometric spreading attenuation rate ($b = -0.5$) and $Q(f)$ models the departure from this rate.

Example Inversion

To illustrate this procedure, an example is given using data from the M4.7 Sparks earthquake (EQID 90) recorded in Region 1 (the Gulf Coast). Figure 3 shows a map of the earthquake...
epicenter and the recording stations used in this analysis. The 2-dimensional S-wave radiation pattern at $f = 1.5$ Hz for this earthquake is shown by the dashed line. At the same frequency, this data is processed as described previously (site effects based on frequency and $V_{s30}$, and radiation pattern effects based on the frequency and azimuthal variation of the 2-dimensional radiation pattern). Figure 3 shows these radiation pattern correction factors for $f = 1.5$ Hz versus azimuth (right), where the small black symbols are the 2-dimensional estimations of the total S-wave motion radiation amplitude pattern normalized by the average over the whole focal sphere, and the red circles are the recording stations. At each stage, the attenuation of the data is fit to Equation 6, as shown in the bottom right of Figure 3. This procedure is repeated for multiple frequencies to estimate the frequency dependence of the apparent anelastic attenuation, $c(f)$, and the apparent $Q(f)$ on an event-by-event basis (e.g. Figure 4) and these results are combined to create the regional models.

The Sparks earthquake, shown in Figure 3, occurred within the CNA region but was well-recorded in the Gulf Coast region. Using this example, for the Gulf Coast region $Q(f)$ analysis, all of the available data within the Gulf Coast region was used (after applying the distance filter on the selection) regardless of the source-to-station travel distances through the CNA region. The Gulf Coast region analysis therefore includes some source-to-station paths with a significant amount of travel in the CNA region, and some with considerably less. For a given distance, one might expect less attenuation of the ground motions at a station with most of the travel path in the CNA, compared to a station with most of the travel path in the Gulf Coast. This will contribute to increased scatter in the attenuation of the ground motions with distance (e.g. Figure 3) and would result in an increase in the standard error of the $c$ coefficient estimates ($se_c$). Any biases introduced by this effect (for all events) are not considered in the analysis and a more elegant treatment of the data selection is an area for future improvement.
Figure 4. Results from the M4.7 Sparks earthquake (EQID 90) with data recorded in Region 1. Top: the mean apparent anelastic attenuation coefficient, $c$ (filled circles), versus frequency, with standard error of the coefficient (triangles). Bottom: the apparent $Q(f)$. The mean (filled circles) and standard error (triangles) are given along with the mean fit (solid line) and 95% confidence intervals for the mean fit (dashed lines). $Q_0$ and $\eta$ with their standard errors are given in the figure.

**RESULTS**

Within each region, apparent $Q(f)$ is estimated independently for each event. Results formatted similarly to Figure 3 and 4 are provided in Appendix B for each of the four regions analyzed and for each event. Appendix B also includes tables summarizing the estimated $Q(f)$ parameters and their uncertainties for each event.

As described previously, the inversion procedure is also performed on each of $EAS_{raw}$, $EAS_{Site}$, $EAS_{Rad}$, and $EAS_{RadSite}$ in order to assess the effectiveness of these data corrections on modeling apparent $Q(f)$. Additionally, the procedure is repeated for each of the three alternative linear site amplification models. The final data correction method selection is based
on reductions in $\sigma_{\text{Resid}}$ and $\sigma_{\text{ec}}$ relative to the uncorrected data. Appendix C shows these reductions, both event-based and averaged over all events within a region, for each site amplification model and NGA-East region. Based on this assessment, the radiation pattern and site adjusted ground motions ($EAS_{\text{RadSite}}$) perform best when using the Stewart et al. (2017) site amplification model in the Gulf Coast and Appalachian regions, and using the Harmon et al. (2019) model in the Central North America region. The radiation pattern adjustment has generally weaker influence on improving the attenuation modeling than the site response adjustment. The radiation pattern adjustment also has large variability in its effectiveness, as manifested by occurrences of increases in $\sigma_{\text{Resid}}$ for some events and decreases in others. This is likely an indication that the simple algorithm used for the radiation pattern effect is too generic and is a matter with room for improvement in future studies.

**Regional Models**

To develop a model for each region, the mean of the event-based estimates of apparent $Q(f)$ within each region is calculated. Figure 5 shows the mean $Q(f)$ (circles) with standard deviations (triangles) for the three regions. The best fit of the mean to the form $Q_{\text{region}}(f) = Q_0 f^\eta$ is also shown. The regional model parameters from this fit are listed in Table 1.

<table>
<thead>
<tr>
<th>Region</th>
<th>Region Name</th>
<th>$Q_0$</th>
<th>$\sigma_{Q_0}$</th>
<th>$\eta$</th>
<th>$\sigma_{\eta}$</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>Gulf Coast</td>
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<td>0.03</td>
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<tr>
<td>2</td>
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<tr>
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<td>Appalachian Province</td>
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<td>0.55</td>
<td>0.05</td>
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</tbody>
</table>
Figure 5. Results for (a) the Gulf Coast, (b) the CNA, and (c) the Appalachian Province showing the $\bar{Q}$ ($f$) (filled circles) and standard deviations (triangles) of the event-based results. The mean fit (solid line) with 95% confidence intervals (dashed lines) are shown. Values of $Q_0$ and $\eta$ are given in each panel.

Discussion

There is general agreement that tectonically stable regions are usually described by higher $Q$ ($f$) and weaker frequency dependence ($\eta$), while active regions are typically characterized by
lower $Q(f)$ and stronger frequency dependence (e.g. Dreiling et al., 2014; Baqer and Mitchell, 1998; Cramer, 2012). Baqer and Mitchell (1998) attributed these differences to the greater amounts of interstitial crustal fluids in western North America. Further, Baqer and Mitchell (1998) found lowest $Q_0$ is in the western United States, with intermediate values in the area spanning from the Colorado Plateau to the Rocky Mountains and in the southern portion of the Atlantic Coastal Plain and the Gulf Coast, with the highest $Q_0$ in the Appalachians. These trends are generally consistent with the results given in Table 1, although the differences between regions in Table 1 are modest. Based on this analysis, $Q_0$ is lowest in the Gulf Coast region ($Q_0 = 278$) and larger, but similar, in the Central North America region ($Q_0 = 465$) and the Appalachian Province ($Q_0 = 451$). The strongest frequency dependence is found in the Gulf Coast ($\eta = 0.60$), and slightly lower frequency dependence is observed in the Appalachian Province ($\eta = 0.55$) and Central North America ($\eta = 0.56$).

Model Comparison

The models developed here are compared with a selection of published models in Figures 6 through 8. For each model, the corresponding geometric spreading coefficient ($b$ value) is given in the figure legend. Two alternative Gulf Coast models are given in Figure 6; Cramer (2012) and Siva et al. (2002). Several models for the CNA and ENA are given in Figure 7 because they are plentiful: Cramer (2012) for CNA, Erickson et al. (2004) for CNA, Cramer (2012) for ENA, Atkinson and Boore (1995), Atkinson and Boore (2014), Boatwright and Seekins (2011), Atkinson (2004), and Erickson (2004) for ENA. Finally, one alternative model for the Appalachian region is presented in Figure 8; Shi et al. (1996).

Of the three Gulf Coast models compared in Figure 6, the one developed in this study has the mildest frequency dependence ($\eta = 0.60$ compared with $\eta = 0.75$ and 0.84). However, as discussed previously the geometric spreading and anelastic attenuation terms can trade off with each other. The Silva et al. (2002) geometric spreading model is magnitude dependent; the value shown ($b = -0.55$) corresponds to $M_5$ and is associated with a steeper attenuation with distance. This will lead to larger $\eta$, and therefore higher $Q(f)$ (lower damping) at high frequencies, to counterbalance this attenuation. The current study value of $Q_0 = 278$ is between the $Q_0$ of the other two models ($Q_0 = 270$ and 351).
Figure 6. Comparison of $Q(f)$ models for the Gulf Coast region (Region 1)

Figure 7 compares two models for CNA and six models for ENA with the results from this study. The CNA region used in this study (Region 2, as defined by PEER NGA-East; Dreiling et al., 2014) contains the region others have described as the ENA (which generally includes northeastern United States and Southeastern Canada, but not central North America) as well as Central North America. With the exception of Boatwright and Seekins (2011; $Q_0 = 410$) the value of $Q_0 = 465$ from the present study is low compared with the other models, with values generally falling between 500 and 700. The slope parameter $\eta = 0.56$ from this study is also consistent with the range of other models.

Figure 8 compares the Shi et al. (1996) model for the Appalachian Province with the results from this study. The current model ($Q_0 = 451, \eta = 0.55$) has steeper slope and is lower at low frequencies than the Shi model ($Q_0 = 573.5, \eta = 0.465$), which results in similarity between the models at frequencies above about 10 Hz. The Shi et al. (1996) model does not assume any specific geometric spreading function because their $Q(f)$ is determined by fitting the spectral shape of $L_g$ wave displacement amplitude spectra (Shi et al. 1996). In general, the proposed model has lower $Q(f)$ at low frequencies, but it is overall in good agreement with the Shi et al. (1996) model.
Figure 7. Comparison of $Q(f)$ models for the CNA or ENA regions, as noted in the legend (Region 2).

Figure 8. Comparison of $Q(f)$ models for the Appalachian Province region (Region 3).

**Evidence for other forms of $Q(f)$**

There is an apparent “flattening” (as phrased here, in frequency space) of $Q(f)$ between approximately 1-5 Hz in some of the results. This observation is not present in the $Q(f)$ of every event, nor is it present in the regional averages, but is common enough in the event-based results to warrant discussion. These observations suggest that the linear form of $Q(f)$ may not be the most appropriate in all situations. Atkinson (2004) proposed a polynomial form for $Q(f)$ in the ENA to accommodate her observation that the $Q(f)$ reached a minimum at 1 Hz and...
increased for lower and higher frequencies. Later, in Atkinson (2012), she found that an exponential form is preferable for $f > 1$ Hz because it is more stable at high frequencies. The flattening is also evident in results from other studies but is not often discussed or modeled, presumably because the form $Q(f) = Q_0 f^n$ has become standard due to its wide application and because there is no theoretical basis for a more complex model. One notable modeling exception is Dave Boore’s stochastic model software (SMSIM; Boore, 2005) which allows for a trilinear $Q$ vs. $f$ model. At present, it is not clear if this is a feature related to the anelastic attenuation or if it comes from other source or site factors, therefore the simpler linear model is adopted. Figure 9 presents a few examples of this behavior from authors of other empirical $Q(f)$ studies in different regions.
Figure 9. A collection of figures with results from other empirical $Q(f)$ studies. (a) Figure from Cramer (2012) showing apparent $Q(f)$ in the Gulf Coast from the Slaughterville earthquake. (b) Figure from Atkinson and Boore (2014) showing multiple event-based apparent $Q(f)$ in eastern North America. (c) Figure from Erickson et al. (2004) showing multiple event-based apparent $Q(f)$ in the central United States. (d) Figure from Atkinson (2004) showing a polynomial fit to $Q(f)$ in eastern North America. (e) From this study, apparent $Q(f)$ in the Appalachian Province from the Jefferson earthquake.
Events with Data in Multiple Regions

Overall, the apparent $Q(f)$ event-based results support the Dreiling et al. (2014) regionalization. Table 2 lists the earthquakes analyzed that have data in multiple regions, along with the $Q_0$ and $\eta$ estimates for each. This comparison allows for testing the boundaries by identifying different apparent $Q(f)$ in multiple regions from the same earthquake. Eleven of these events have data in both Region 1 (Gulf Coast) and Region 2 (CNA). Qualitatively, the results are as expected and support the region boundaries because $Q_0$ is larger for the CNA data than for the Gulf Coast data, and $\eta$ values are smaller for the CNA as compared with the Gulf Coast. In fact, all eleven events have larger $Q_0$ for the CNA than for the Gulf Coast. For comparing the other two regions, only the Mineral earthquake (EQID88) had data in both the Appalachian Province (Region 3) and in the Atlantic Coastal Plain (Region 4). The results for this single event are also consistent with expectations; the estimated $Q_0$ for the Appalachians is larger with less frequency dependence than for the Atlantic Coast.

Two earthquakes are identified as good candidates for further testing of the sensitivity of apparent $Q(f)$ within the Gulf Coast region (Table 3Error! Reference source not found.). These are the Guy and Greenbrier earthquakes. Both events have enough data both in the northernmost section of the Gulf Coast region (Memphis area) and to the west (Texas area) that the analysis can be repeated for each sub-region separately. Figure 10 shows the division of the data and the results of this analysis for the M4.7 Greenbrier earthquake. Table 3 summarizes the event-based $Q(f)$ estimates for each sub-region. In both cases, the $Q_0$ estimate is larger and the $\eta$ estimate is smaller for the northern Mississippi Embayment (Memphis region) compared with the Texas region. The conclusiveness of these results would benefit from more data than just two events. This region is a good candidate for potential refinement with respect to attenuation models and should be studied further.
Table 2. Comparison of results for earthquakes with data in multiple regions.

<table>
<thead>
<tr>
<th>EQID</th>
<th>Earthquake Name</th>
<th>State/Province</th>
<th>M</th>
<th>Hyp. Depth (km)</th>
<th>Eqk. Region</th>
<th>Region 1 $Q_0$</th>
<th>Region 1 $\eta$</th>
<th>Region 2 $Q_0$</th>
<th>Region 2 $\eta$</th>
<th>Region 3 $Q_0$</th>
<th>Region 3 $\eta$</th>
<th>Region 4 $Q_0$</th>
<th>Region 4 $\eta$</th>
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<td>5.3</td>
<td>15.7</td>
<td>2</td>
<td>331</td>
<td>0.48</td>
<td>902</td>
<td>0.26</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
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<td>Mt Carmel Aftershock</td>
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<td>14.0</td>
<td>2</td>
<td>224</td>
<td>0.87</td>
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<td>0.2</td>
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<td>-</td>
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</tr>
<tr>
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<td>Slaughterville</td>
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<td>14.0</td>
<td>2</td>
<td>251</td>
<td>0.76</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>Arkansas</td>
<td>3.86</td>
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<td>2</td>
<td>216</td>
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<tr>
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<td>Bethel Acres</td>
<td>Oklahoma</td>
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<td>2</td>
<td>251</td>
<td>0.64</td>
<td>852</td>
<td>0.34</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
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<td>Guy</td>
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<td>1286</td>
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<td>Virginia</td>
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<td>-</td>
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<td>90</td>
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<td>Oklahoma</td>
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<td>723</td>
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<td>Oklahoma</td>
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Table 3. Comparison of results within Region 1.

<table>
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<tr>
<th>NGA East EQI D</th>
<th>Earthquake Name</th>
<th>State/Province</th>
<th>M</th>
<th>Hyp. Depth (km)</th>
<th>Eqk. Region</th>
<th>Texas (W Gulf Coast) Q₀</th>
<th>Texas (W Gulf Coast) η</th>
<th>Memphis (N Gulf Coast) Q₀</th>
<th>Memphis (N Gulf Coast) η</th>
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<td>76</td>
<td>Guy</td>
<td>Arkansas</td>
<td>3.9</td>
<td>5.0</td>
<td>2</td>
<td>137</td>
<td>1.30</td>
<td>278</td>
<td>0.90</td>
</tr>
<tr>
<td>80</td>
<td>Greenbrier</td>
<td>Arkansas</td>
<td>4.68</td>
<td>4.0</td>
<td>2</td>
<td>212</td>
<td>1.05</td>
<td>348</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Figure 10. Comparison of results for the M4.7 Greenbrier earthquake in Region 1, split (a-b) to the southwest and (c-d) to the northeast.
SUMMARY AND CONCLUSIONS

This study uses smoothed FAS data from well-recorded events in the CENA as collected and processed by PEER NGA-East (PEER, 2015) to estimate regional \( Q(f) \) using an assumption of average geometrical spreading applicable to the distance ranges considered, a correction for the radiation pattern effect, and a correction for site response based on \( V_{s30} \). Independent analyses of the data adjusted each of these effects and find that together they improve estimates of \( Q(f) \), with the radiation pattern adjustment having generally weaker influence on the attenuation modeling than the site response. Apparent \( Q(f) \) from multiple events are combined within each region to develop the regional models.

\( Q(f) \) is usually modeled with the form \( Q(f) = Q_0 f^{\eta} \), where \( Q_0 \) is the \( Q \) value at 1 Hz, and \( \eta \) is the slope parameter. Using this form, models are developed for three regions as defined by PEER (Dreiling et al. 2014): The Gulf Coast, Central North America, and the Appalachian Province. There was not sufficient data to adequately constrain the model for a fourth region, the Atlantic Coastal Plain.

The apparent \( Q(f) \) event-based results support the Dreiling et al. (2014) regionalization, as significantly different regional \( Q(f) \) estimates are identified for particular events with data recorded in multiple regions. For two events recorded in the Gulf Coast region with data both in the northernmost Mississippi Embayment (Memphis region) and to the west (Texas area), higher \( Q_0 \) estimates are found in the Memphis region. This region is a candidate for potential refinement with respect to attenuation models in future investigations.

The regional models are consistent with expectations; the tectonically stable regions (CNA, Appalachian Province) are usually described by higher \( Q(f) \) and weaker frequency dependence (\( \eta \)), and the Gulf Coast model is characterized by lower \( Q(f) \) and stronger frequency dependence. The models are suitable for estimating the \( Q(f) \) for all three regions based on a literature review and a comparison with previously published models.
DATA AND RESOURCES

The ground motion data were processed by PEER NGA-East (Goulet et al., 2014) and provided by Christine Goulet (personal communication, 2019). Regression analyses and graphics production were performed using the numeric computing environment MATLAB (www.mathworks.com last accessed 25 June, 2020). Matlab codes for our Q(f) inversion method are available at https://github.com/jay14bay/USGS_G17AP00034

ACKNOWLEDGEMENTS

This project was sponsored by the USGS under award number G17AP00034. Thanks to the Pacific Earthquake Engineering Research Center for making their ground motion databases, including those from NGA-East, publicly available. Thanks to David Boore for providing helpful review comments, to Christine Goulet for her assistance with the PEER NGA-East database, and to Youssef Hashash and Okan Ilhan for providing guidance with their FAS site amplification model. All maps in this report were created using the M_Map Matlab package (Pawłowicz, 2019).

REFERENCES


Bayless 2


### APPENDIX A

Table A-1. Earthquakes used in the estimation of apparent anelastic attenuation.

<table>
<thead>
<tr>
<th>Earthquake Name</th>
<th>State/ Province</th>
<th>Earthquake Date</th>
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<th>Hyp. Depth (km)</th>
<th>Eqk. Region</th>
<th>No. Stats Region 1</th>
<th>No. Stats Region 2</th>
<th>No. Stats Region 3</th>
<th>No. Stats Region 4</th>
<th>No. Stats All Regions</th>
<th>No. Stats Region 1A</th>
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<tr>
<td>Au Sable Forks</td>
<td>New York</td>
<td>April 20, 2002</td>
<td>4.99</td>
<td>10.0</td>
<td>2</td>
<td>0</td>
<td>14</td>
<td>13</td>
<td>0</td>
<td>21</td>
<td>0</td>
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<tr>
<td>Caborn</td>
<td>Indiana</td>
<td>June 18, 2002</td>
<td>4.55</td>
<td>17.5</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>10</td>
<td>3</td>
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<td>Charleston</td>
<td>South Carolina</td>
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<td>9.0</td>
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<td>5</td>
<td>5</td>
<td>11</td>
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<td>Alabama</td>
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<td>4.62</td>
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<td>2</td>
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<td>5</td>
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APPENDIX B

This appendix contains figures summarizing the apparent $Q(f)$ calculation for each event in:

Region 1: Gulf Coast
Region 2: Central North America
Region 3: Appalachian Province
Region 4: Atlantic Coastal Plain

APPENDIX C: $\sigma_{\text{Resid}}$ AND $se_c$ REDUCTIONS

This appendix contains figures summarizing the difference in $\sigma_{\text{Resid}}$ and $se_c$ for the site and radiation pattern corrected data, relative to the uncorrected data (uncorrected – corrected) so that positive values represent improvement in the fit. Figures are shown for each of the four regions and for the three different site response models considered in the analysis.